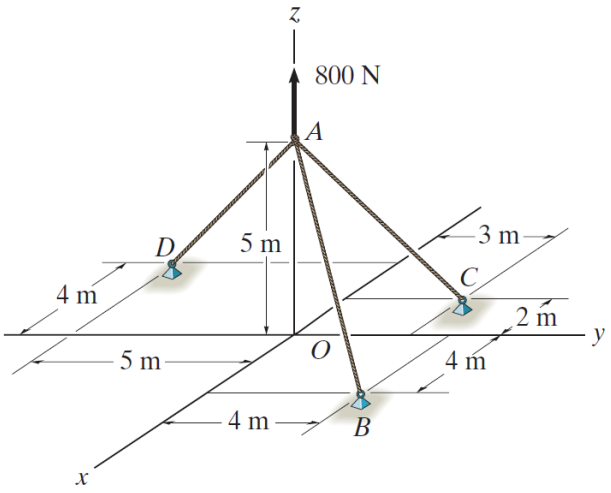
Problem 3-61

Determine the tension in each cable for equilibrium.



Prob. 3–61

Solution

Write position vectors to points A, B, C, and D.

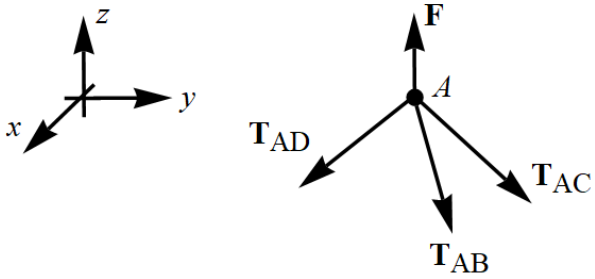
$$\mathbf{r}_A = \langle 0, 0, 5 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 4, 4, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -2, 3, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle -4, -5, 0 \rangle \text{ m}$$

Draw a free-body diagram for the knot at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{F} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

Write out each of the unit vectors and then simplify the left side.

$$T_{\text{AB}}\frac{\langle 4-0,4-0,0-5\rangle}{\sqrt{(4-0)^2+(4-0)^2+(0-5)^2}}+T_{\text{AC}}\frac{\langle -2-0,3-0,0-5\rangle}{\sqrt{(-2-0)^2+(3-0)^2+(0-5)^2}}$$
$$+T_{\text{AD}}\frac{\langle -4-0,-5-0,0-5\rangle}{\sqrt{(-4-0)^2+(-5-0)^2+(0-5)^2}}+F\langle 0,0,1\rangle=\mathbf{0}$$

$$T_{\text{AB}}\left\langle\frac{4}{\sqrt{57}},\frac{4}{\sqrt{57}},-\frac{5}{\sqrt{57}}\right\rangle+T_{\text{AC}}\left\langle-\sqrt{\frac{2}{19}},\frac{3}{\sqrt{38}},-\frac{5}{\sqrt{38}}\right\rangle+T_{\text{AD}}\left\langle-2\sqrt{\frac{2}{33}},-\frac{5}{\sqrt{66}},-\frac{5}{\sqrt{66}}\right\rangle+F\langle 0,0,1\rangle=\mathbf{0}$$

$$\left\langle\frac{4}{\sqrt{57}}T_{\text{AB}}-\sqrt{\frac{2}{19}}T_{\text{AC}}-2\sqrt{\frac{2}{33}}T_{\text{AD}},\right.$$
$$\frac{4}{\sqrt{57}}T_{\text{AB}}+\frac{3}{\sqrt{38}}T_{\text{AC}}-\frac{5}{\sqrt{66}}T_{\text{AD}},$$
$$\left.-\frac{5}{\sqrt{57}}T_{\text{AB}}-\frac{5}{\sqrt{38}}T_{\text{AC}}-\frac{5}{\sqrt{66}}T_{\text{AD}}+F\right\rangle=\langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned}\frac{4}{\sqrt{57}}T_{\text{AB}}-\sqrt{\frac{2}{19}}T_{\text{AC}}-2\sqrt{\frac{2}{33}}T_{\text{AD}}&=0\\ \frac{4}{\sqrt{57}}T_{\text{AB}}+\frac{3}{\sqrt{38}}T_{\text{AC}}-\frac{5}{\sqrt{66}}T_{\text{AD}}&=0\\ -\frac{5}{\sqrt{57}}T_{\text{AB}}-\frac{5}{\sqrt{38}}T_{\text{AC}}-\frac{5}{\sqrt{66}}T_{\text{AD}}+F&=0\end{aligned}\right\}$$

Solving it and plugging in $F=800$ N yields

$$T_{\text{AB}}=\frac{11\sqrt{57}}{115}F\approx 578\text{ N}$$
$$T_{\text{AC}}=\frac{2\sqrt{38}}{115}F\approx 85.8\text{ N}$$
$$T_{\text{AD}}=\frac{2\sqrt{66}}{23}F\approx 565\text{ N}.$$